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# The stability of tensile deformation of single ductile fibre-ductile matrix composites with weak interfaces

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The embedded molybdenum fibre in a copper matrix composite was elongated apparently uniformly although the interface was so weak that necking in the fibre could not be suppressed by the matrix. To explain this result, a possible mechanism was suggested where suppression of necking in the fibre is caused by strain hardening of the composite as a whole and by an increment in strain rate in the cross-section where the fibre starts necking, but the incremental deformation amount and the incremental strain rate of the cross-section are small. On the basis of this mechanism, and with the aid of Hart's criterion, a new instability approach to tensile behaviour of the composite is presented. It was found that the stability of the composite is determined mainly by the strain-hardening exponent of the composite, which is determined by the modified rule of mixtures. The present derived condition of instability of the composite is in good agreement with that proposed by Mileiko.

## 1. Introduction

Ductile fibres embedded in composites exhibit uniform elongation [1–5] or multiple necking [6–8], when the composites are elongated beyond $\epsilon_{fu}$, the strain at which fibres start necking and subsequently fail when tested alone. As a mechanism of uniform elongation, Piehler [4] has pointed out that necking of the fibres is arrested if the interfacial bonding is strong enough, and composites can deform uniformly along the tensile axis beyond $\epsilon_{fu}$. Mileiko [9] and Garmong and Thompson [10] applied the plastic instability approach to the tensile behaviour of composites with strong interfacial bonding in order to predict the failure strain of composites on the assumption that composites can deform until necking begins in composites as a whole, and obtained good results. On the other hand, according to Venett *et al.* [6], Schoene and Scala [7], and the authors [8], multiple necking of fibres in composites requires no strong interfacial bonding. The multiple necking of fibres, which of course yields high ductility in composites, has been explained by local work hardening of the matrix adjacent to the neck; i.e. the strengthened matrix can take load from the fibre until the load-bearing capacity of the composite at this area exceeds the load-bearing capacity elsewhere [6–8].

Although it has been supported that strong interfacial bonding is necessary for uniform elongation of the fibre, we have recently found that embedded fibres are able to be elongated apparently uniformly beyond $\epsilon_{fu}$ even when the interfacial bonding is so weak that debonding possibly occurs at $\epsilon_{fu}$ at the interface [11]. We have also found by using the same weakly bonded composites [11] that the deformation parameters such as flow stress, internal stress, effective stress, strain-hardening exponent, stress exponent of strain rate, effective stress exponent of dislocation velocity and activation volume obey the simple or modified rule of mixtures even in the deformation stage III-(2) ranging from $\epsilon_{fu}$ to the failure of the composite as a whole. Thus the inherent features of the fibre are conserved in stage III-(2), and the parameters of the composite are determined by the inherent parameters of the components. Previous work cannot explain why the fibre in the


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weakly bonded composite could be elongated apparently uniformly beyond $\epsilon_{fu}$, maintaining its inherent features and when such a composite becomes instable.

The aims of the present paper are to discuss the stability of weakly bonded composites on the basis of the above observations. The observations were made using a single thick fibre composite with two components: an outer component of matrix and a core of fibre. Therefore no effects due to neighbouring fibres or due to small fibre strenghening of the matrix need enter the discussion [4, 8, 12–14].

## 2. Experimental procedure

The fibre and matrix employed were molybdenum wire of 500 $\mu$m diameter and copper, respectively. The preparation methods are described in the previous paper [11]. Two types of fibre were employed. The ultimate tensile strength, $\sigma_{fu}$, and true strain at ultimate loading, $\epsilon_{fu}$, of one type (described as type A) were 1.03 GPa and 0.08, respectively. This type was the same as the fibre used in the previous work [11] Another type (described as type B) had $\sigma_{fu} = 0.78$ GPa and $\epsilon_{fu} = 0.16$.

Some specimens were annealed at 873 K for $1.8 \times 10^3$ sec. The other specimens were thermally cycled 100 times between 273 and 573 K to reduce interfacial strength [11]. The annealed composites containing type A and B fibres are referred to in future as type A and B composites, and the thermally cycled and then annealed composites containing type A and B fibres are described as type TA and TB composites, respectively. The differences in $\epsilon_{fu}$ and $\sigma_{fu}$ between type A and TA and those between type B and TB fibres were negligible. The only difference between type A and TA and that between type B and TB composites was interfacial shear strength [11]. The interfaces of all types were very weak in tension, being very near zero [11].

The strain-hardening exponent and strain at ultimate loading were measured by a simple tension test and the stress exponent of strain rate by a strain-rate cycling test.

## 3. Results

### 3.1. Strain-hardening exponent

The strain-hardening exponent $n$, is defined by

$$n = \partial \ln \sigma / \partial \ln \epsilon, \tag{1}$$

where $\sigma$ and $\epsilon$ are true stress and true strain at $\sigma$, respectively. The modified rule of mixtures of the strain-hardening exponent of composites, $n_c$, is given by [11]

$$n_c = n_f \alpha + n_m(1-\alpha) \tag{2}$$

where the subscripts c, f and m refer to composite, fibre and matrix, respectively, and $\alpha = \sigma_f V_f / (\sigma_f V_f + \sigma_m V_m)$. By substituting into Equation 2 the simultaneously values of $\sigma_f$ and $\sigma_m$ for $\epsilon < \epsilon_{fu}$, and the measured values of $\sigma_m$ and the values of $\sigma_f$ inferred by extrapolating the $\sigma_c - V_f$ relation to $V_f = 1.00$ [11] for $\epsilon > \epsilon_{fu}$, we calculated $\alpha$. By plotting the measured values of $n_c$ against $\alpha$, it was confirmed that the measured values of $n_c$ of all specimens obeyed Equation 2 both in the ranges of $\epsilon < \epsilon_{fu}$ and $\epsilon > \epsilon_{fu}$, as ascertained already [11]. The $n_c$ of type A and TA composites at $\epsilon = 0.06$

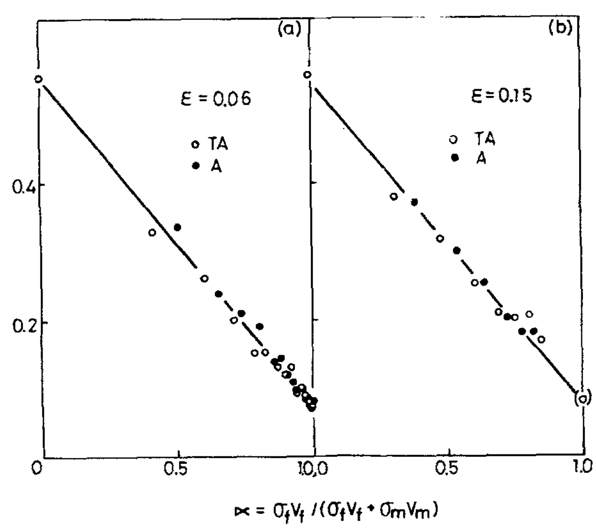


*Figure 1* The measured values of $n_c$ of type A and TA specimens plotted against $\alpha = \sigma_f V_f / (\sigma_f V_f + \sigma_m V_m)$.

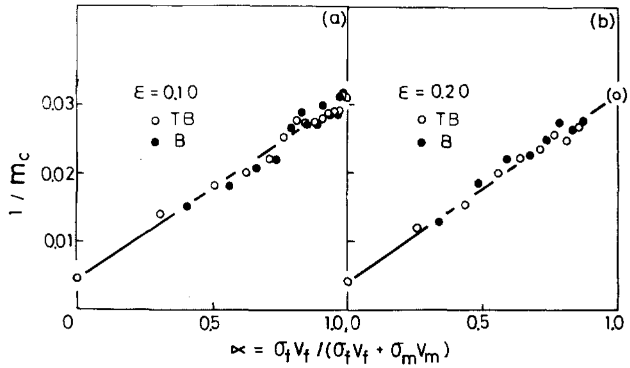


*Figure 2* The measured values of $1/m_c$ of type B and TB specimens plotted against $\alpha = \sigma_f V_f / (\sigma_f V_f + \sigma_m V_m)$.

and 0.15 is typically shown in Fig. 1 in which $n_c$ varies linearly with varying $\alpha$.

### 3.2. Stress exponent of strain rate

The stress exponent of strain rate, $m$, is defined by

$$m = \partial \ln \dot{\epsilon} / \partial \ln \sigma \quad (3)$$

where $\dot{\epsilon}$ is the strain rate. The rule of mixtures of the stress exponent of strain rate of the composite, $m_c$, is given by [11]

$$1/m_c = \alpha/m_f + (1-\alpha)/m_m \quad (4)$$

where $\alpha = \sigma_f V_f / (\sigma_f V_f + \sigma_m V_m)$. The measured values of $1/m_c$ also varied linearly as a function of $\alpha$, as typically shown in Fig. 2 where the measured values of $1/m_c$ of type B and TB composites are plotted against $\alpha$.

### 3.3. Strain at ultimate loading

The measured values of the strain at ultimate loading (necking strain) of the composites, $\epsilon_{cu}$, are shown in Fig. 3.

## 4. Discussion

In the previous paper [11], it was confirmed that, in the weakly bonded composites, the deformation parameters such as flow stress, internal stress and effective stress obey the simple rule of mixtures, and the strain-hardening exponent, stress exponent of strain rate, effective stress exponent of dislocation velocity and activation volume obey the modified rule of mixtures not only for $\epsilon < \epsilon_{fu}$, but also for $\epsilon > \epsilon_{fu}$. Thus the inherent nature of the fibre is conserved beyond $\epsilon_{fu}$, and the deformation parameters of the composite are determined by the inherent parameters of fibre and matrix. This implies that the mechanical interaction between the components or local inhomogeneous deformation in the composite is of a small order. In fact, the embedded fibres exhibited apparently uniform elongation but no inhomogeneous deformation such as multiple necking for $\epsilon > \epsilon_{fu}$.

To explain the uniform elongation and high ductility of the embedded fibres, we must consider

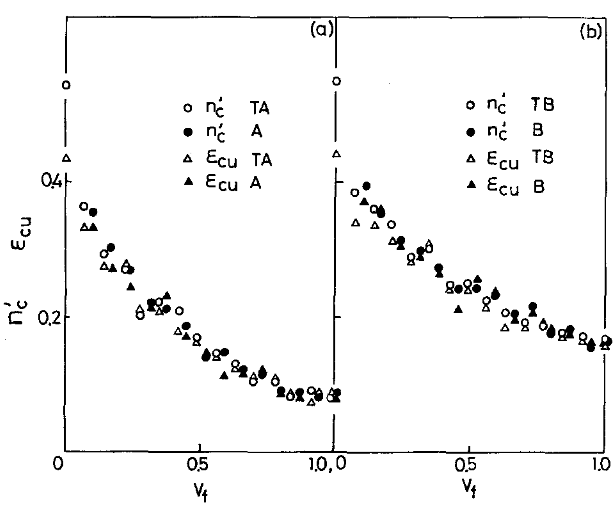


*Figure 3* The measured values of $\epsilon_{cu}$ of type A, TA, B and TB specimens versus $V_f$. The measured values of $n'_c$ of the same specimens are superimposed, to show the correlation of $n'_c$ and $\epsilon_{cu}$.

two possible mechanisms. One is the constraint effect of the matrix to arrest necking in the fibre [4]. This mechanism, however, should be disputed, since the interfacial strength of the present composites was very weak. Another possible mechanism, according to which we would like to develop a theory on the instability of composites, is that the suppression of necking in the fibre is caused by the strain hardening of the composite as a whole, and by an increment in strain rate in the cross-section where the fibre starts necking, when the composite is elongated to $\epsilon_{fu}$. Judging from the experimental results that the deformation in the composite, to a first approximation, proceeds homogeneously for $\epsilon > \epsilon_{fu}$, the incremental deformation amount of the cross-section where the fibre starts necking is small, and the development of the necking in the fibre is inhibited at once. In other words, at $\epsilon = \epsilon_{fu}$, a small portion of the length of the specimen has a cross-section that differs from the cross-section of the remainder by a small amount, and the loss of load bearing capacity due to necking in the fibre is compensated for at once by the strain hardening of the fibre and matrix and the increment in strain rate. However, the deformation amount at a cross-section where the fibre starts necking is so small that the fibre is seen to deform apparently uniformly.

As the interaction between the components is possibly negligible for $\epsilon > \epsilon_{fu}$ [11], the applied load $P_c$ on the composites is given by

$$P_c = \sigma_f A_f + \sigma_m A_m, \tag{5}$$

where $A$ is the cross-sectional area at any distance $l$. Since Equation 5 must hold simultaneously at all points $l$, the variation of $P_c$ as a function of $l$ must be zero. Thus

$$0 = dP_c/dl = A_f(d\sigma_f/dl) + \sigma_f(dA_f/dl) + A_m(d\sigma_m/dl) + \sigma_m(dA_m/dl). \tag{6}$$

Also, $d\sigma/dl$ is given by

$$d\sigma/dl = (\partial\sigma/\partial\epsilon)(d\epsilon/dl) + (\partial\sigma/\partial\dot{\epsilon})(d\dot{\epsilon}/dl). \tag{7}$$

We also need the relationships

$$\partial\sigma/\partial\epsilon = (\sigma/\epsilon)(\partial \ln \sigma/\partial \ln \epsilon) = \sigma_n/\epsilon \tag{8}$$

$$\partial\sigma/\partial\dot{\epsilon} = (\sigma/\dot{\epsilon})(\partial \ln \sigma/\partial \ln \dot{\epsilon}) = \sigma/m\dot{\epsilon} \tag{9}$$

and

$$dA/dl = -A(d\epsilon/dl). \tag{10}$$

Substituting Equations 7 to 10 into Equation 6, we have

$$\begin{aligned} dP_c/dl = {} & A_f\,\{(\sigma_f n_f/\epsilon_f)(d\epsilon_f/dl) + (1/\dot{\epsilon}_f)(\sigma_f/m_f)(d\dot{\epsilon}_f/dl)\} \\ & + A_m\,\{(\sigma_m n_m/\epsilon_m)(d\epsilon m/dl) + (1/\dot{\epsilon}_m)(\sigma_m/m_m)(d\dot{\epsilon}_m/dl)\} \\ & + (-\sigma_f A_f)(d\epsilon_f/dl) + (-\sigma_m A_m)\, d\epsilon_m/dl) = 0. \end{aligned} \tag{11}$$

Judging from the experimental results, the fibre and the matrix are, to a first approximation, subjected to the same strain and strain rate, and the volume fraction of fibre remains constant at any $l$. Then we put

$$\epsilon_f = \epsilon_m = \epsilon \tag{12}$$

$$\epsilon_f = \epsilon_m = \epsilon \tag{13}$$

$$A_f/(A_f + A_m) = V_f \tag{14}$$

$$A_m/(A_f + A_m) + V_m. \tag{15}$$

Substituting Equations 12 to 15 into Equation 11, we have

$$\begin{aligned} & \{(n_f\sigma_f V_f + n_m\sigma_m V_m)/\epsilon - (\sigma_f V_f + \\ & + \sigma_m V_m)\}(d\epsilon/dl) + (1/\dot{\epsilon})(\sigma_f V_f/m_f + \\ & + \sigma_m V_m/m_m)(d\dot{\epsilon}/dl) = 0. \end{aligned} \tag{16}$$

In order to know whether or not the composite becomes unstable at $\epsilon_{fu}$, we apply the stable criterion of Hart [15] to the composite

$$(d\dot{A}_c/dA_c)_{P_c} \leqslant 0 \tag{17}$$

where $A_c$ is the cross-sectional area of composite, and $\dot{A}_c$ is $dA_c/dt$. To combine Equation 17 with Equation 16, we obtain $d\dot{A}_c/dA_c$ as follows, $\dot{\epsilon}$ and $d\epsilon$ are given by

$$\dot{\epsilon} = -\dot{A}_c/A_c \tag{18}$$

and

$$d\epsilon = -dA_c/A_c, \tag{19}$$

respectively. By modifying Equation 18, we have

$$d\dot{\epsilon} = (-A_c d\dot{A}_c + \dot{A}_c dA_c)/A_c^2. \tag{20}$$

Combining Equation 18 to 20, we have

$$d\dot{A}_c/dA_c = (d\dot{\epsilon} - \dot{\epsilon}d\epsilon)/d\epsilon = d\dot{\epsilon}/d\epsilon - \dot{\epsilon} \tag{21}$$

Combining Equation 16 with Equation 21, we

$$\left(\frac{\mathrm{d}\dot{A}_c}{\mathrm{d}A_c}\right)_{P_c} = \dot{\epsilon}\left[\frac{\sigma_f V_f + \sigma_m V_m - \dfrac{n_f\sigma_f V_f + n_m\sigma_m V_m}{\epsilon} - \left(\dfrac{\sigma_f V_f}{m_f} + \dfrac{\sigma_m V_m}{m_m}\right)}{\dfrac{\sigma_f V_f}{m_f} + \dfrac{\sigma_m V_m}{m_m}}\right]. \tag{22}$$

At $\epsilon = \epsilon_{fu} = n_f (< n_m)$, $(\mathrm{d}\dot{A}_c/\mathrm{d}A_c)_{P_c}$ is negative, indicating that the composite is stable at the strain at which necking would occur in the fibre when tested separately.

The strain at which the composite becomes unstable, $\epsilon_{cu}$, may be calculated by setting $(\mathrm{d}\dot{A}_c/\mathrm{d}A_c)_{P_c} = 0$, as

$$\epsilon_{cu} = \frac{n_f\sigma_f V_f + n_m\sigma_m V_m}{\sigma_f V_f + \sigma_m V_m} \times \left(1/\left[1 - \frac{(\sigma_f V_f/m_f) + (\sigma_m V_m/m_m)}{(\sigma_f V_f + \sigma_m V_m)}\right]\right)$$

$$= \frac{n_f\alpha + n_m(1-\alpha)}{1 - \left(\dfrac{\alpha}{m_f} + \dfrac{1-\alpha}{m_m}\right)}. \tag{23}$$

According to Equation 4, $\alpha/m_f + (1-\alpha)/m_m$ is equal to $1/m_c$, which is smaller than 0.032 for the present composites as typically shown in Fig. 2. Therefore Equation 23 is reduced to

$$\epsilon_{cu} = n_f\alpha + n_m(1-\alpha) \tag{24}$$

The $\epsilon_{cu}$ given by Equation 24 is the same as $n_c$ given by Equation 2. In other words, $\epsilon_{cu}$ is, to a first approximation, determined by the modified rule of mixtures given for $n_c$.

To examine Equation 24, we measured the $n_c$ value in the deformation stage prior to necking (denoted $n'_c$ in future) and compared them with the measured values of $\epsilon_{cu}$. Experimentally we measured the $n'_c$ by using the equation $n'_c = \ln(\sigma_2/\sigma_1)/\ln[(\epsilon_{cu} - 0.02)/(\epsilon_{cu} - 0.04)]$ where $\sigma_2$ and $\sigma_1$ are true stresses of the composite at $\epsilon = \epsilon_{cu} - 0.02$ and $\epsilon_{cu} - 0.04$, respectively. The measured values of $n'_c$ were superimposed in Fig. 3. The measured values of $n'_c$ were in good agreement with those of $\epsilon_{cu}$ with the exception of the range of small $V_f$ and matrix. The discrepancy between $n'_c$ and $\epsilon_{cu}$ in the samples with small $V_f$ and matrix could be attributed to the fact that the $n_m$ value was constantly 0.55 for $\epsilon < 0.40$ but decreased with increasing strain for $\epsilon > 0.40$. Although the reason why $n_m$ decreased for $\epsilon > 0.40$ was not known in this work, we can conclude that $n'_c$ is in good agreement with $\epsilon_{cu}$ for the range of constant $n_m$, excluding the range of $\epsilon > 0.40$ where $n_m$ varied.

Next we will show that Equation 24 may be derived from Mileiko's theory [9]. Assuming that the true stress–strain curve is expressed in the form

$$\sigma = F(\epsilon)^n, \tag{25}$$

where $F$ is a constant, and assuming that the expression of Equation 25 is valid beyond $\epsilon_{fu}$ for the fibre, the normal stress of the composite $\sigma_c^n$ at any $\epsilon$ is given by

$$\sigma_c^n = \{F_f(\epsilon)^{n_f} V_f + F_m(\epsilon)^{n_m} V_m\} \exp(-\epsilon). \tag{26}$$

Differentiating Equation 26 with respect to $\epsilon$, and setting $\mathrm{d}\sigma_c^n/\mathrm{d}\epsilon = 0$, we have the strain at the maximum loading $\epsilon_{cu}$,

$$\mathrm{d}(\sigma_c^n/\mathrm{d}\epsilon)_{\epsilon=\epsilon_{cu}} = 0$$

$$= n_f F_f V_f(\epsilon_{cu})^{n_f - 1} + n_m F_m V_m(\epsilon_{cu})^{n_m - 1} - F_f V_f(\epsilon_{cu})^{n_f} - F_m V_m(\epsilon_{cu})^{n_m}. \tag{27}$$

Substituting $F_f(\epsilon_{cu})^{n_f} = (\sigma_f)_{\epsilon=\epsilon_{cu}}$ and $F_m(\epsilon_{cu})^{n_m} = (\sigma_m)_{\epsilon=\epsilon_{cu}}$ into Equation 27, we have Equation 28 which is the same as Equation 24,

$$\epsilon_{cu} = n_f\alpha + n_m(1-\alpha) \tag{28}$$

where

$$\alpha = (\sigma_f)_{\epsilon=\epsilon_{cu}} V_f/[(\sigma_f)_{\epsilon=\epsilon_{cu}} V_f + (\sigma_m)_{\epsilon=\epsilon_{cu}} V_m.$$

Mileiko assumed in his theory that the strength of the fibre–matrix interface was sufficient to prevent the fibre necking. However, the assumption was not introduced into his mathematical procedure. The assumption used in his mathematical procedure was that the embedded fibre deforms uniformly beyond $\epsilon_{fu}$ in the composite. Comparing the present result with Mileiko's theory, we can say that his theory is valid if the embedded fibre undergoes apparently uniform elongation whether the interfacial bonding is strong or weak.

It should be noted that, in general, it is not possible to express the stress–strain curve exactly for the whole deformation range by the simple form as in Equation 25, and in some cases, $n$ varies during deformation and does not correctly correspond to necking strain. For such a case, further investigation should be carried out.

## 5. Conclusions

The embedded fibre in copper matrix composites prepared by a plating method was elongated apparently uniformly although the interface was so weak that the necking in the fibre seemed unable to be suppressed by the matrix. To account for this result, a new instability approach was presented to the tensile behaviour of the composite. From the present approach, it was predicted that the stability of the composite is determined mainly by the strain-hardening exponent of the composite as a whole, which is determined by the modified rule of mixtures. The prediction was verified experimentally. The present derived condition of the instability of the composite is in good agreement with that proposed by Mileiko.

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